January 26, 2018

MATHEMATICS 218 QUIZ I

NAME LE

Time: 55 minutes Spring 2017-18

Circle your section number:

Michella Bou-Eid			Sabine El Khoury			Giussepe Della Sala			Hazar Abu-Khuzam			Rana Nassif		
. 1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
8 M	11 M	2 M	12 M	3 M	4 M	8 F	11 M	11 F	2 T	3:30 T	5 T	1 M	3 M	4 M

PROBLEM GRADE

PART I

- 1 ----- / 12
- 2 ----- / 10
- 3 ----- / 16
- 4 ----- / 12

PART II

5	6	7	8.	9	10	11	12
a	a	a	a	a	·a	а	a
b	(b)	b	b	b	b	(b)	b
0	С	0	0	0	С	С	С
d	d	d	d.	d	d	d	d
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5-12 ----- / 32

PART III

13	14	15	16	17	18
T	(1)	T	T		CD
(F)	F	(F)	(F)	F	F

13-18 ----- / 18

TOTAL

----- / 100

PART I. Answer each of the following problems in the space provided for each problem (Problem 1 to Problem 4).

1. Find the values of k for which the following system

$$x +2y -kz = 0$$

 $-x +3y +k^2z = k$
 $-5y -2z = 1$

has

- a. a unique solution
- b. no solution
- c. infinitely many solutions.

a) Unique Solution 5 $k^{2}-k-2 \neq 0$ $(k-2)(k+1) \neq 0$ $(k+2) k \neq -1$

b) no solution: $k^2 = 2 = 0$ and $k+1 \neq 0$ (k-1)(k+1) = 0 and $k \neq 1$ (k-1)(k+1) = 0

So [k=2].

I infinitely many solutions k-k-2=0 kend k+1=0 k=2, k=-1 k=2

[12 points]

2. Let A be a 3x3 matrix such that
$$(A^{-1}2D)^{-1}$$
 $\begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$. Find A.

$$\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix} \xrightarrow{R_1 \oplus R_3} \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \xrightarrow{R_1 \oplus R_3} \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \xrightarrow{R_1 \oplus R_3} \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

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$$= \begin{pmatrix} 0 & 1$$

$$A = \left(\begin{array}{rrr} 1 & 1 & 2 \\ -2 & -1 & -3 \\ 1 & 2 & 3 \end{array}\right),$$

Col. (1)
$$+$$
 Col(2) $=$ Col(3)

[3 points]

$$\Rightarrow |A| = 0$$

$$\Rightarrow A = 6 \text{ not invertible}$$
or $Show = det(A) = 0$
(b) For a real number a, consider the vector $\mathbf{b} = \begin{pmatrix} 1 \\ a-2 \\ a^2+a \end{pmatrix}$. Find the values of a such that

the linear system AX=b (with X=
$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$
) is consistent.

[7 points]

(c) Let a=1 in (b). Find all the solutions $X = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$ of the linear system AX=b.

[6 points]

 $x_{1} + x_{2} + 2x_{3} = 1$ $x_{3} = t = f_{1} = variable$ $x_{2} + x_{3} = 1$ $x_{2} + x_{3} = 1$ $x_{3} = 1 - x_{3} = 1 - t$ $x_{2} - 2x_{3} = 1 - 1 + t - 2t = -t$

4. (a) Let B be a
$$3\times3$$
 skew-symmetric matrix ($B^T = -B$). Prove that det (B)=0

BT = B \Rightarrow |BT = B = (-1)³, |B| $\xrightarrow{3\times3}$ [6 points]

 \Rightarrow |B| = -|B|

 \Rightarrow |A| = -|B|

 \Rightarrow |A| = -|B|

 \Rightarrow |A| = -|B|

(b) Let A be a 2×2 matrix such that $A^2+A-I=0$. Find A^{-1} and prove that A cannot be skew-symmetric.

Skew-symmetric.

A² + A = I

A (A+I) = I

So A is inverted and

A = A+I

Suppose A is skew symmetric 2x2 matrix

$$\Rightarrow A = \begin{pmatrix} 0 & \alpha \\ \alpha & 0 \end{pmatrix} = \begin{pmatrix} 0 & -\alpha \\ \alpha & 0 \end{pmatrix} = \begin{pmatrix} 0 & -\alpha \\ \alpha & 0 \end{pmatrix} = \begin{pmatrix} 0 & -\alpha \\ \alpha & 0 \end{pmatrix}$$

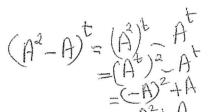
But $A = A + I = \begin{pmatrix} 0 & \alpha \\ \alpha & 0 \end{pmatrix} + \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & -\alpha \\ \alpha & 1 \end{pmatrix}$

So $\begin{pmatrix} 1 & \alpha \\ \alpha & 1 \end{pmatrix} = \begin{pmatrix} 0 & -\alpha \\ \alpha & 0 \end{pmatrix} + \begin{pmatrix} 1 & 0 \\ \alpha & 1 \end{pmatrix} = \begin{pmatrix} 0 & -\alpha \\ \alpha & 1 \end{pmatrix}$

So $A = A + I = \begin{pmatrix} 0 & -\alpha \\ \alpha & 0 \end{pmatrix} + \begin{pmatrix} 0 & -\alpha \\ \alpha & 1 \end{pmatrix} = \begin{pmatrix}$

PART II. Circle the correct answer for each of the following multiple choice problems (Problem 5 to Problem 12) IN THE TABLE IN THE FRONT PAGE. [4 points for each correct answer, NO PENALTY FOR A WRONG ANSWER IN THIS PART].

- 5. Let A be a 3×3 skew-symmetric ($A^T=-A$) matrix. Which one of the following statements is
- (a) A^T is skew symmetric
- (b) $(A^T A)$ is skew symmetric
- $(c)(A^2-A)$ is symmetric
- (d) $a_{11} = a_{22} = a_{33} = 0$
- (e) $A^T A$ is symmetric



[4 points]

The value(s) of k for the system with corresponding augmented matrix $\begin{bmatrix} 1 & 2 & 1 & 0 \\ 0 & 1 & 1 & k \end{bmatrix}$ to be 6.

consistent are

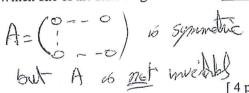


- (b) k=-1
- (c) $k \neq 1$
- (d) $k \neq -1$
- (e) none of the above



[4 points]

- Let A be an nxn symmetric matrix. Which one of the following statements is FALSE
- (a) A^T is symmetric
- (b) 3AA^T+A²+2I is symmetric.
- (c) A is invertible
- (d) A A^T is symmetric



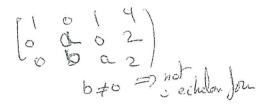
- 8. Let A be a square matrix such that A3=A. Then
- (a) A is not invertible
- (b) $det(A) = \pm 1$
- (Ĉ) If A is invertible then A-1=A
- (d) $A^7=A^2$ (e) None of the above

Let A be a matrix of the form

$$A = \left(\begin{array}{cccc} 1 & 0 & 1 & 4 \\ 0 & a & 0 & 2 \\ 0 & b & a & 0 \end{array}\right)$$

Which one of the following statements is TRUE?

- (c) If b is not 0, then A is not in row echelon form.
- d. If a=0 and b=1, then A is in row echelon form.
- e. none of the above



[4 points]

10. Let A be a 3x3 matrix such that det(A)=2. Which one of the following statements is TRUE?

a.
$$det(A-2I)=0$$
.

$$6 \det(A^{t}) = \det(A^{4}).$$

- c. AtA is not invertible.
- d. $det(2A) = det(A^2)$.
- e. none of the above

[4 points]

- 11. Let A be an n×n matrix. Which one of the following statements is **FALSE**:
- If AX=0 has only the trivial solution, then A²X=0 has only the trivial solution.
- (b). If A≠0, then the matrix equation AX=b has a unique solution for all b.
- c. If AX=b has infinitely many solutions for some b then A is not invertible.
- d. If det(A) is not 0 then the reduced row echelon form of A is I.

[4 points]

12. Let
$$\mathbf{v_1} = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$$
, $\mathbf{v_2} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$ and $\mathbf{v_3} = \begin{pmatrix} 3 \\ 5 \\ 2 \end{pmatrix}$. Which one of the following statements is

- (a) If $v_3 v_2 = cv_1$, then c=2.
- (b) $2v_3 4v_1 = 2v_2$
- (c) The only values of c,d such that $cv_1 + dv_2 = v_3$ are c=2, d=1.
- (d) There exists a real number c such that $v_3 cv_2 = 0$.

$$\binom{3}{5}$$
 \pm $c \binom{1}{0}$

[4 points]

<u>PART III</u>. Answer TRUE or FALSE only, <u>IN THE TABLE IN THE FRONT PAGE</u> (3 points for each correct answer, and <u>-1 point penalty for each wrong answer</u>)

